LITERATURE CITED

1. M. A. Styrikovich, Z. L. Miropol'skii, et al., in: Heat and Mass Transfer [in Russian], Vol. 2, Part 1, Nauka i Tekhnika, Minsk (1972).
2. D. B. Spalding, Convective Mass Transfer [Russian translation], Énergiya, Moscow (1965).
3. V. I. Adamovskii, S. S. Kutateladze, and L. S. Stokolov, Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1967).
4. G. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill (1969).
5. Z. L. Miropol'skii, R. I. Shneerova, and A. I. Karamysheva, in: Fourth International Conference on Heat Transfer, Paris, Paper B4.7 (1970).
6. V. E. Doroshchuk and F. P. Lantsman, Teploénergetika, No. 12 (1970).
7. A. P. Ornatskii and A. M. Kichigin, Teploénergetika, No. 2 (1961).
8. A. P. Ornatskii and A. M. Kichigin, Teploénergetika, No. 6 (1962).
9. A. P. Ornatskii, Teploenergetika, No. 3 (1963).
10. S. S. Kutateladze, Fundamentals of the Theory of Heat Transfer [in Russian], Nauka, Novosibirsk (1970).
11. K. Goldman, H. Firstenberg, and T. Lombardi, Teploperedacha, No. 2 (1961).
12. L. S. Tong, Intern. J. Heat Mass Transfer, 11, No. 7 (1968).

SOME PROBLEMS OF MEASUREMENT OF THERMAL CONDUCTIVITY BY THE METHOD OF COAXIAL CYLINDERS

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An instrument for determining the thermal conductivity of liquids by the method of coaxial cylinders is described. The main attention is devoted to methodological problems of measurements of the thermal conductivity of liquids by this method. Experimental results on the thermal conductivity water obtained using this equipment are presented.

In recent years the method of coaxial cylinders has been extensively used in thermophysical measurements. Several types of measuring units, operating both in absolute and relative variants, have been made. However, the methodological problems of measuring the thermal conductivity have not been discussed in sufficient detail in many studies. This is largely true in regard to the problems of considering the nonuniformity of the temperature field along the length of the inner cylinder, the temperature distribution in the transverse cross section of the unit, the heat loss, and the radiative component of the heat flux.

The measuring unit described below was developed by us for the investigation of thermal conductivity of liquid solutions in a wide range of variation of the state parameters.

The measuring unit (Fig. 1) was developed according to the absolute method of coaxial cylinders. It was made of refined copper. Its operating surfaces were chrome-plated and polished. A hole of 6 mm diameter was drilled along the axis of the inner cylinder (1), in which an electric heater in a steel Kh18N10T sheath (2) was placed. The heater was a ceramic tube 2 mm in diameter, on which Constantan wire of 0.15 mm diameter was wound; the wire was coated by silk insulation impregnated by high-temperature lacquer. The step of the spiral winding of the heater was 0.35 mm, which made it possible to produce a uniform heat flux in the inner cylinder of the unit. The construction of the heater and the protective sheath almost eliminated unheated segments at the end faces of the cylinder.

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The inner cylinder was mounted coaxially in the outer cylinder (3) with the help of centering devices. The centering objects (4) with 1.5 mm diameter are made of textolite and are fixed to micrometric screws (5). Caps (7) 25 mm in length are placed at the end face of the outer cylinder; these caps are provided with apertures (8) for filling the cell with the investigated substance. The end gap between the caps and the inner cylinder is maintained with the use of textolite spacers (6). The length of the measuring cells together with the end caps equals 250 mm.

For measuring the temperature difference in the layer of the investigated substance occupying the gap between the coaxial cylinders and the temperature of the experiment we used three-junction differential Nichrome—Constantan thermocouples and single-junction copper—Copel thermocouples. The thermal electrodes of the thermocouples were made from high-temperature lacquer and were placed in protective sheaths made of steel Kh18N10T, which in turn were placed in specially drilled holes in the cylinders. The thermocouple junctions were placed at the midpoint along the length of the cylinders and were fixed in the protective sheaths with the use of copper inserts. Thus, in the present construction of the measuring cells the thermocouples and the electric heaters are not in contact with the investigated medium and are not subjected to the action of pressure. In order to decrease the overflow of heat along the protecting sheaths annular holes (9) are placed in the zone of their exit from the inner cylinder and textolite plugs are inserted in these holes.

The power supply to the heater of the measuring cells was obtained from a P-136T dc stabilizer. The current intensity and the voltage drop in the heater were measured by the usual potentiometric scheme.

The measuring cell is placed in an autoclave (10) in which the packing is accomplished by gasket (11). The autoclave with the cell is placed in a liquid thermostat in which the temperature is automatically maintained constant with an accuracy of ±0.02°C. The pressure in the autoclave is transmitted through a circulating Teflon dividing tank with the use of a hydraulic press and was measured by a standard manometer.

The thermal-conductivity coefficient of the investigated substances was computed from the formula

$$\lambda = \frac{Q}{A(\Delta t_{\text{meas}} - \Delta t_{\text{equip}})\beta} . \qquad (1)$$

A set of inner cylinders of different diameters offers the possibility of conducting measurements of the thermal-conductivity coefficient of the investigated substances in annular gaps equal to 0.696, 0.487, and 0.238 mm.

An analysis of the heat field of the cells shows that in the general case the distribution of the heat flux generated by the heater occurs in two ways. This can be written in the following manner:

$$Q = \lambda (t_1 - t_2) \left\{ 2\pi \left[\frac{L}{\ln \frac{D_2}{D_1}} + \frac{D_1^2}{4\delta_e} \right] \right\} . \qquad (2)$$

The expression contained in the curly brackets is the geometric constant A of the instrument.

For obtaining the exact value of the cell constant it is necessary to remember that the simple division of the total heat flux into two terms does not take into consideration the fraction of the heat flux arriving at the end-face corners of the cell. The change in the configuration of the gap in the zone of the end-face angles causes a redistribution of the heat field in this region. The pattern of the thermal field (as for the plane layers, since the measuring gap is small compared to the diameter of the inner cylinder) in the zone of the end-face angles is shown in Fig. 2; this is obtained by simulating this region of the measuring cell on an integrator of type ÉGDA-9/60. The figure shows that the homogeneity of the thermal field is disturbed in this zone, but the distorting effect of the end-face angles disappears at distances of (2-3)δ on both sides of the tip of the end-face angle. Therefore, it can be concluded that the distribution of the thermal field in a measuring cell of finite dimensions is almost the same as the field of the end-face part for the same region under the assumption of infinite length of the cylinder. This fact offers the possiblity

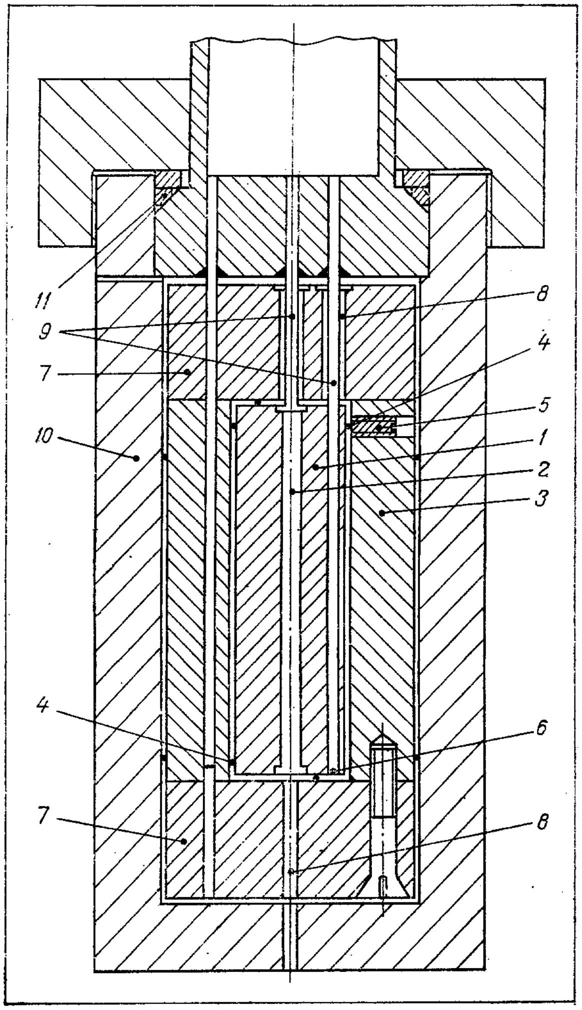

Fig. 1

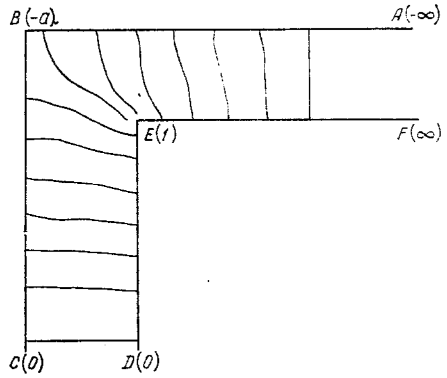

Fig. 2

Fig. 1. Schematic diagram of the measuring cell.

Fig. 2. Distribution of heat flux at the end faces of the measuring cell.

of using the Christoffel–Schwartz transformation for a quantitative determination of the perturbed region of the heat flux in the zone of the end-face angles and permits one to refine the instrument constant. In accordance with Fig. 2 the transformation is written in the following form:

$$\frac{dZ}{dW}=\frac{K}{W}\left(\frac{W-1}{W+a}\right)^{\frac{1}{2}}. \tag{3}$$

Integrating (3) with the use of the substitution $\beta = [(W-1)/(W+a)]^{1/2}$, we obtain

$$Z=K\left\{\ln\frac{\sqrt{W+a}+\sqrt{W-1}}{\sqrt{W+a}-\sqrt{W-1}}-\frac{2}{\sqrt{a}}\operatorname{arctg}\sqrt{\frac{W-1}{W+a}a}\right\}. \tag{4}$$

The integration constant vanishes as a result of the choice of the coordinate origin at the point E. For determining quantities K and a we put $W = -a$; then we have

$$a=\left(\frac{\delta_e}{\delta_c}\right)^2;\ K=\frac{\delta_e}{\pi}.$$

Applying the logarithmic transformation of type $\pi W_1 = \ln W$ to expression (4), we can take into consideration the perturbed part of the heat flux in the zone of the end-face angles through the increment in the geometrical dimensions of the cell. These increments are, respectively, equal to

$$\Delta D = 2\left\{\frac{2\delta_e}{\pi\delta_c}\ \text{arctg}\ \frac{\delta_c}{\delta_e} + \frac{1}{\pi}\ \ln\frac{\delta_c^2+\delta_e^2}{4\delta_e}\right\}\delta_c,$$
$$\Delta L = 2\left\{\frac{2\delta_c}{\pi\delta_e}\ \text{arctg}\ \frac{\delta_e}{\delta_c} + \frac{1}{\pi}\ \ln\frac{\delta_c^2+\delta_e^2}{4\delta_c}\right\}\delta_e. \quad (5)$$

Then the geometrical constant of the instrument with the heat fluxes in the zones of the end-face angles taken into consideration can be expressed in the following way:

$$A = 2\pi\left\{\frac{L+\Delta L}{\ln\frac{D_2}{D_1}} + \frac{(D_1+\Delta D)^2}{4\delta_e}\right\}. \quad (6)$$

Making use of the analogy between thermal and electric fields, the cell constant can be obtained from the measurement of the capacity of the condenser formed by the coaxial cylinders:

$$A = \frac{C}{\varepsilon_v \varepsilon_a}. \quad (7)$$

Considering that the cell constant obtained with the use of the capacity measurement is more accurate, below in the computational equation (1) we use the value of A computed from formula (7).

The realization of the method of radial heat flux presumes the absence of heat loss across the end-face surfaces of the inner cylinder; this is possible for infinitely long cylinders or in the presence of ideal adiabatic conditions at the end faces of the cell. The use of measuring cells of finite dimensions in practice leads to the result that the presence of the end-face component of the heat flux has a significant effect on the formation of the temperature field of the inner cylinder. In order to consider the nonisothermicity of the surface of the inner cylinder in the computational equation (1) we introduce a coefficient β. The determination of coefficient β by an analytical method leads to the problem of nonsymmetric temperature distribution along the length of the inner cylinder due to different heat losses through the upper and lower end faces of the cylinder. Using the principle of superposition of potential fields, the general solution of the nonsymmetric problem was obtained in the form of a sum of partial solutions of two symmetric problems. The solution of the first problem permitted us to obtain the temperature field distribution along the length of the inner cylinder due to the heat losses from the end faces through the layer of the investigated substance and along the centering bodies. The second solution determines the change in the temperature field of the cylinder due to the heat escape along the protective sheaths.

The starting point in the determination of the temperature field distribution is the differential equation of heat balance for an element of length dx of the inner cylinder. In setting up this equation the radial temperature drop in the cylinder was disregarded due to the high thermal conductivity of copper and the temperature of the outer cylinder was taken as the origin for the temperature readings. Taking these assumptions into consideration, the differential equation of heat balance is of the form

$$\frac{Q}{L}\,dx = \left\{-\lambda_c F_c\,\frac{dt}{dx} - \left[-\lambda_c F_c\,\frac{d}{dx}\left(t - \frac{dt}{dx}\,dx\right)\right]\right\} + \frac{2\pi\lambda t}{\ln\frac{D_2}{D_1}}\,dx. \quad (8)$$

The left-hand side of this equation determines the amount of heat released by element dx, while the right-hand side characterizes the amount of heat arriving along the cross section of the cylinder toward the end face and transmitted by conduction across the layer of the investigated substance.

After some transformations we obtain a linear differential equation of the type

$$\frac{d^2t}{dx^2} - At + B = 0, \qquad (9)$$

where

$$A = \frac{2\pi\lambda}{\lambda_c F_c \ln \frac{D_2}{D_1}}; \quad B = \frac{Q}{\lambda_c F_c L}.$$

The choice of the coordinate origin in the solution of Eq. (9) is determined by the corresponding boundary conditions. Putting the coordinate origin at the point lying on the axis of the inner cylinder at a distance L/2 from its end faces, we write the boundary conditions in the following form:

$$\begin{gathered} \left.\frac{dt}{dx}\right|_{x=0} = 0, \\ -\lambda_c F_c \left.\frac{dt}{dx}\right|_{x=\frac{L}{2}} = \frac{\lambda F_e}{\delta_e} t + \frac{4\lambda_p F_p}{\delta} t. \end{gathered} \qquad (10)$$

Solving Eq. (9) with boundary conditions (10), we obtain

$$t = \frac{B}{A} - \frac{\frac{B}{A}\left(\frac{\lambda F_e}{\lambda_c F_c \delta_e} + \frac{4\lambda_p F_p}{\lambda_c F_c \delta}\right)}{\sqrt{A}\,\mathrm{sh}\left(\sqrt{A}\,\frac{L}{2}\right) + \left(\frac{\lambda F_e}{\lambda_c F_c \delta_e} + \frac{4\lambda_p F_p}{\lambda_c F_c \delta}\right)\mathrm{ch}\left(\sqrt{A}\,\frac{L}{2}\right)}\,\mathrm{ch}\left(\sqrt{A}x\right), \qquad (11)$$

where x = 0-L/2.

The first term in the right-hand side of (11) characterizes the temperature at the surface of the inner cylinder in the absence of heat losse, while the second term characterizes the magnitude of the change in the temperature field due to the heat loss from the end faces and through the centering bodies.

The determination of the relationship which would permit one to determine the change in the temperature field of the inner cylinder due to heat escape along the protective sheaths presupposes a knowledge of the amount of heat passing through their base. For reducing the problem to the axisymmetric problem we replace the effect of two protective sheaths on the formation of the temperature field by one equivalent sheath with parameters λ_{eq} and F_{eq}.

The amount of heat passing through the base of the equivalent capillary is determined from the well-known expression

$$q = \lambda_{eq} F_{eq} b \Delta t \,\mathrm{cth}\, bl,$$

where

$$b = \sqrt{\frac{2\pi\lambda_i}{\lambda_{eq} F_{eq} \ln \frac{d_0}{d_{eq}}}}.$$

Then putting the coordinate origin at the point lying on the axis of the inner cylinder of length 2L at a distance L from its end faces and writing the boundary conditions in the form

$$\left.\frac{dt}{dx_1}\right|_{x_1=0}=0,$$
$$-\lambda_c F_c \left.\frac{dt}{dx_1}\right|_{x_1=L}=\lambda_{eq}F_{eq}b\Delta t \operatorname{cth} bl, \tag{12}$$

we obtain the solution of Eq. (10) for boundary conditions (12):

$$t=\frac{B}{A}-\frac{\dfrac{\lambda_{eq}F_{eq}}{\lambda_c F_c}b\Delta t \operatorname{cth} bl}{\sqrt{A}\operatorname{sh}(\sqrt{A}L)}\operatorname{ch}(\sqrt{A}x_1), \tag{13}$$

where x_1 = 0–L. The second term in (13) characterizes the distortion of the temperature field due to the heat loss along the sheaths.

Taking the sum of the second terms of Eqs. (11) and (13), we obtain an expression which permits one to determine the magnitude of distortions of the temperature field of the inner cylinder introduced by heat losses from the end faces through the layer of the investigated substance, along the centering bodies at the protective sheaths, and along the conduits:

$$t=\frac{B}{A}-\left\{\frac{\dfrac{B}{A}\left(\dfrac{\lambda F_e}{\lambda_c F_c \delta_e}+\dfrac{4\lambda_p F_p}{\lambda_c F_c \delta}\right)}{\sqrt{A}\operatorname{sh}\left(\sqrt{A}\dfrac{L}{2}\right)+\left(\dfrac{\lambda F_e}{\lambda_c F_c \delta_e}+\dfrac{4\lambda_p F_p}{\lambda_c F_c \delta}\right)\operatorname{ch}\left(\sqrt{A}\dfrac{L}{2}\right)}\times\right.$$
$$\left.\times\operatorname{ch}(\sqrt{A}x)+\frac{\dfrac{\lambda_{eq}F_{eq}}{\lambda_c F_c}b\Delta t \operatorname{cth} bl}{\sqrt{A}\operatorname{sh}(\sqrt{A}L)}\operatorname{ch}(\sqrt{A}x_1)\right\}, \tag{14}$$

$$x=0-\frac{L}{2};\ x_1=0-L.$$

The dependence of the relative magnitudes of the change of the temperature field of the inner cylinder on its length, obtained with the use of (11) for δ_c = 0.696 mm, is shown in Fig. 3a. This figure shows that the magnitude of the distortions of the temperature field introduced by the heat loss from the end faces and along the centering bodies is determined by the thermal conductivity of the investigated substance.

The successive determination of the distribution of the temperature field of the cylinder for toluene for δ_c = 0.696 mm is shown in Fig. 3b. Curve 1 gives the nature of the temperature distribution along the length of the cylinder due to heat losses along the protective sheaths; curve 2 characterizes the relative magnitude of the change in the temperature field of the cylinder computed from Eq. (11); curve 3, which characterizes the total amount of distortions, can be obtained from Eq. (14).

It is obvious that the distribution of the temperature field is a function of the magnitudes of the cylindrical and end-face gaps, the thermal-conductivity coefficient of the investigated substance, and the ratio L/D_1. The relationship between the relative magnitude of the distortions at the center of the inner cylinder for different gaps δ_c is shown in Fig. 3c. The dashed curve 1 characterizes the effect of the protective sheath on the magnitude of the temperature field at the center of the cell, computed from Eq. (14) for δ_c = 0.696 mm. Similar dependences 3 and 4 are obtained for δ_c = 0.487 and 0.238 mm, respectively. Curve 2 characterizes the magnitude of the distortions of the temperature field at the center of the cell and is obtained with the use of (11) for δ_c = 0.696 mm. The analysis showed that the effect of the end faces, the centering bodies, the sheaths, and the conduits decreases with the increase in λ of the investigated substances and with the decrease of the cylindrical gap.

Using formula (14), which permits one to determine the temperature distribution along the length of the inner cylinder, we can determine the coefficient β:

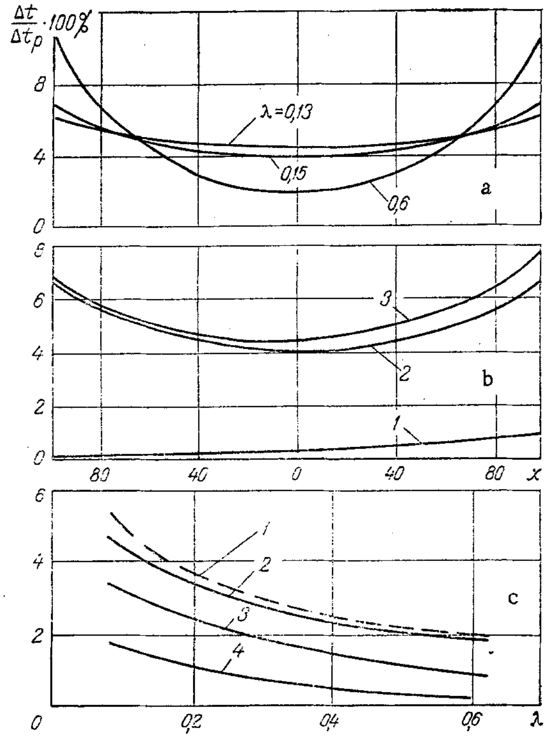

Fig. 3. Dependence of the relative magnitude of the change in the temperature of the inner cylinder on its length: a) the temperature distribution along the length of the cylinder for different λ, [W/(m•°K)]; b) temperature distribution along the length of the cylinder due to heat loss from the end faces of the cylinder for toluene; c) change in the temperature at the center of the cylinder for different λ [W/(m• °K)] and δ_c (mm).

$$\beta = \frac{\frac{2\pi L}{\ln D_2/D_1}\,\Delta\bar{t}_c + \frac{2F_e}{\delta_e}\,(\Delta\bar{t}_c' + \Delta\bar{t}_e'')}{\frac{2\pi L}{\ln\frac{D_2}{D_1}} + \frac{2F_e}{\delta_e}}. \quad (15)$$

The junctions of the differential thermocouple, measuring the temperature difference in the layer of the investigated substance, are placed in the protective sheaths, which in turn are placed in the drilled holes in the body of the inner and outer cylinders. Since the temperature difference of the surfaces of the cylinders occurs in the computational equation (1), there is a need for correcting the measured value Δt_{meas}. Usually, in such cases a correction for the thermocouple "mounting," which formally takes into consideration the temperature drop in the wall, is introduced. The temperature distribution caused by the inhomogeneities of the cylinders in the zone of the measurements of the temperature difference is not taken into consideration. It is obvious that due to the large difference in the thermal-conductivity coefficients of copper and stainless steel the radial heat flux undergoes changes. This distortion of the radial temperature field in the zone of location of the thermocouple junctions can result in a general redistribution of the thermal field, which is reflected in the value of the temperature at the surface of the cylinder. In order to obtain the true temperature distribution in the transverse cross section of the measuring cells, this region is investigated with the use of an ÉGDA-9/60 integrator. The model was prepared in accordance with the requirements imposed in simulating potential fields on electrically conducting paper and represented the investigated region of the cell on 20:1 scale without any distortions; its external diameter was 800 mm. The boundary conditions were realized with the use of special clamp-strips made of bakelite insulation foil.

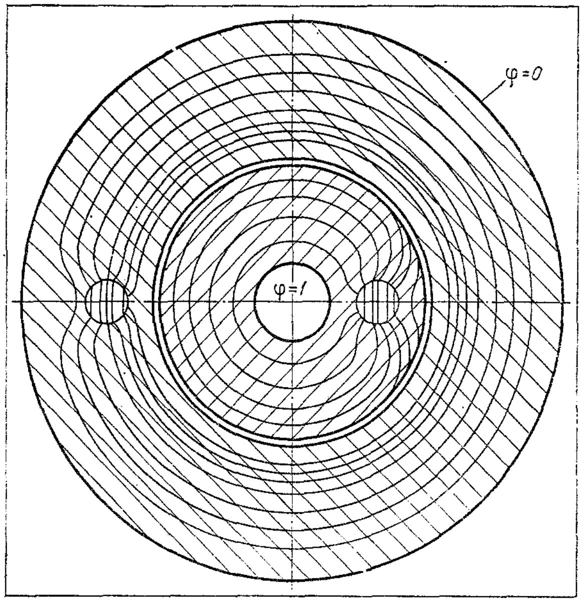

Fig. 4. A temperature distribution in the transverse cross section of the cell.

The model of the transverse cross section of the measuring cell is shown in Fig. 4; the applied potentials and the temperature field obtained from electrical simulation are also shown. As seen from the figure, for the inner cylinder the isotherms in the region of location of the thermocouple sheath are deformed, which in turn is reflected in the displacement of the thermal axis relative to the geometric center of symmetry. This displacement of the thermal axis of the isotherms passing through the thermocouple junction leads to a decrease of the magnitude of the correction for the thermocouple mount in comparison with the usual methods of computing Δt_{equip}. For the outer cylinder, in spite of the analogous pattern of the temperature distribution, the consideration of the perturbed part of the heat flux does not lead to any quantitative change in the magnitude of the equipment correction. The perturbing action of the container of the inner cylinder not only leads to a redistribution of the thermal field in the zone of thermocouple joints, but (as follows from Fig. 4) it also disturbs the isothermicity along the generating surface of the cylinder. Therefore, in experiments for determining the thermal-conductivity coefficient of water for δ_c = 0.696 mm the magnitude of the equipment correction decreases by 40% compared to the computed value obtained by the usual method. It should be noted that the magnitude of the distortions of the temperature field, caused by the presence of the containers, is determined by δ_c, the ratio λ_C/λ_c, and the location and the diameter of the container. As shown by the investigations in the study of the temperature distribution for different cylindrical gaps, the largest distortions appear for water and the distortions increase with the decrease of δ_c.

Using electrical simulation we investigate the problem of estimating the "temperature waves" which are inherent to spiral-type heating elements.

The results of the investigation obtained with the use of an ÉGDA-9/60 integrator offer the possibility of obtaining a quantitative estimate of such waves. The isotherms near the cross section of the loop of the wire of the heater have a form which is very nearly circular. As the distance from the heater increases, the isotherms become deformed; one of them becomes lemniscate, passing through the points at which the total heat flux vanishes. A quantitative estimate shows that at a distance equal to three diameters of the heater wire the isotherms degenerate into straight lines parallel to the axis of the heater. Therefore, the chosen construction permits one to assume that the heat flux emanating from the heater is uniform.

The thermal conductivity of water was investigated with a measuring gap of 0.487 mm; the following results were obtained: For t of 29.5, 46.5, 58.7, 72.4, 85.7, 100.4, and 112.6°C, λ is equal to 0.617, 0.641, 0.654, 0.667, 0.675, 0.682, and 0.686 W/(m·°K), respectively.

NOTATION

Q, amount of heat produced by electrical heater; A, geometric constant of measuring unit; Δt_{meas}, measured temperature difference; Δt_{equip}, correction allowing for arrangement of thermocouple junctions; β, coefficient allowing for nonisothermicity of internal cylinder; λ, thermal conductivity of test substance; D_1, external diameter of internal cylinder; D_2, inside diameter of external cylinder; δ_c, magnitude of cylindrical gap; δ_e, magnitude of end gap; L, length of internal cylinder; λ_c, thermal conductivity of internal cylinder material; λ_p, thermal conductivity of material of centering bodies; F_c, cross-sectional area of internal cylinder; F_p, cross-sectional area of centering bodies; λ_{eq}, equivalent thermal conductivity, F_{eq}, cross-sectional area of equivalent capillary; λ_i, thermal conductivity of insert material; d_0, inner diameter of conduit; d_{eq}, equivalent diameter of capillary; l, length of capillary; x, distance from the middle of cylinder to the place of thermocouple junction fixing; x_1, distance from the middle of doubled cylinder to the place of thermocouple junction fixing; C, electric capacity; ε_a, dielectric constant of air; ε_v, dielectric constant in vacuum; φ, potential; Δt_{calc}, calculated temperature difference, taken equal to 1 °K in analysis of Eqs. (11), (13), and (14).

LITERATURE CITED

1. E. Schmidt and W. Sellschopp, Forshung auf dem Gebiete des Ingenierwesens, 3, 277 (1932).
2. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed., Oxford University Press (1959).
3. M. A. Lavrent'ev and B. V. Shabat, Methods of the Theory of Functions of Complex Variables [in Russian], Fizmatgiz, Moscow (1958).
4. W. Leidenfrost, Intern. J. Heat Mass Transfer, 7, 477 (1964).
5. P. F. Fil'chakov and V. I. Panchishin, ÉGDA Integrators. Simulation of Potential Fields on Electrically Conducting Paper [in Russian], Izd. Akad. Nauk UkrSSR, Kiev (1961).